

STATEMENT REPORT

under the procedure for acquisition
of the educational and scientific degree “Doctor”
in Professional Field 4.5. Mathematics
(Operations research),
At Sofia University “St. Kliment Ohridski”,
Faculty of Mathematics and Informatics
Candidate: Boyan Kolev Stefanov
Reviewer: Prof. Nadezhda Kostadinova Ribarska, DSci
May 10, 2024

I am writing this review in my capacity as a member of the scientific jury, according to Order No. RD-38-129/1.03.2024 of the Rector of Sofia University. The presented dissertation “Optimal Control Problems under Uncertainty” (Задачи на оптималното управление в условие на неопределеност) consists of 91 pages and is written in English. It contains an introduction, three chapters, a conclusion and a bibliography. An abstract in Bulgarian and an abstract in English (the first one 39 pages long and the second one 46 pages long), as well as all other documents required by the procedure (including a report from the anti-plagiarism system) are submitted. I accept for review all submitted materials.

I have known Boyan for more than five years – first as a student, then as a master’s and doctoral student, participant in the optimization seminar. I know him as a good colleague and a motivated mathematician.

The dissertation is devoted to the study of dynamic games on infinite time horizon. This research is important from both a theoretical and a practical point of view, and requires serious technical skills.

In the first two chapters of the dissertation, linear-quadratic dynamic games on an infinite interval in the presence of control constraints are considered (for continuous time in chapter 1 and for discrete time in chapter 2). The problems are very similar, but the techniques employed are extremely different. An extensive literature has been devoted to linear-quadratic dynamic games, but there are no results in which simultaneously the time interval is infinite and control constraints are present. It is proved that, under natural assumptions on the problem data, for each pair of admissible controls the trajectory of the game tends to zero and, therefore, from a certain point onwards the constraints on the control of the first player are not active. The idea is to decompose the

game into two time intervals – finite and infinite, and to follow different strategies in each of the two cases. Sufficient conditions for optimality are also obtained. Considerable difficulties, both conceptual and technical, have been overcome. Two interesting examples (together with respective numerical simulations) are presented in the dissertation and in the related publications. I would like to note that in some statements open-loop strategies are used and other statements are concerned with closed-loop strategies. The rules of the game should be strictly formulated.

In the third chapter, a result of Aseev, Velov and Krastanov from 2017 is used, in which the adjoint variable is defined explicitly. A Pontryagin maximum principle-type necessary condition for “weakly overtaking Nash equilibrium” (as far as I know, this concept was introduced in the thesis) in the context of a discrete zero-sum game on an infinite time horizon is derived. Also, under suitable convexity conditions, it is proved that the necessary optimality condition is also sufficient for “weakly overtaking optimality” in the discrete-time optimal control problem.

The results of the dissertation have been published (or submitted for publication) in four articles. Two of these are out of print (one in *Dynamic Games and Applications*, Q3 and one in *Lecture Notes in Computer Science*, SJR), one has been accepted for publication in *Lecture Notes in Computer Science*, and one has been submitted for publication. Two of the papers are joint with the scientific supervisor, and the other two - with the scientific supervisor and Dr. Rosen Rosenov. I am convinced that Boyan Kolev’s contribution to these publications is significant.

The publications based on Boyan Stefanov’s dissertation exceed a lot the minimum national requirements (according to Art. 2b, paras. 2 and 3 of the RSARB) and, accordingly, the additional requirements of SU “St. Kliment Ohridski” for the acquisition of an educational and scientific degree “doctor” in professional field 4.5 Mathematics (Operations Research).

Part of the results of the dissertation were reported at four prestigious international scientific conferences and repeatedly at the IMI-BAS and FMI-SU reporting sessions, as well as at the optimization seminar. I have personally attended many of these presentations and I have witnessed the growth of Boyan Kolev as a lecturer.

The abstracts (in Bulgarian and in English) accurately and comprehensively reflect the results, described in the dissertation.

A remark. The English language of the dissertation is very good, but the Bulgarian language in the abstract is bad and full of errors.

The dissertation contains original results. Referencing other people’s re-

sults is comprehensive and correct. The presentation is good. The dissertation work contains scientific results that are an original contribution to the scientific field and which can be developed in future.

I confirm that the presented dissertation and the scientific publications related to it meet the requirements of ZRASRB, the Regulations for its application and the relevant Regulations of SU “St. Kliment Ohridski” for the candidate’s acquisition of the educational and scientific degree “doctor” in professional field 4.5 Mathematics. In particular, the candidate satisfies the minimum national requirements in the professional field and no plagiarism has been found in the scientific papers submitted.

Based on the above, I recommend the scientific jury to award Boyan Kolev Stefanov the educational and scientific degree “Doctor” in Scientific Area 4. Natural sciences, Mathematics and Informatics, Professional field 4.5. Mathematics (Operations Research).

10.05.2024

(Prof. N. Ribarska, DSci)